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Optimization of ΔV Earth-Gravity-Assist Trajectories

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An analysis of delta-velocity Earth-gravity-assist (ΔV-EGA) trajectories is carried out by using an indirect optimization method. In comparison to previous literature, the eccentricity of the Earth's orbit is taken into account, and simplifying assumptions, concerning the position and direction of velocity impulses, are removed. However, the focus is on the application of the theory of optimal control to complex interplanetary missions and ΔV-EGA trajectories are only illustrative examples. In particular, it is shown that the optimization of this kind of mission, according to the patched-conic approximation, can be obtained by considering only the two-body problem equations in the heliocentric frame. The necessary optimum conditions for the Earth flyby are extended to consider a minimum-height constraint, which is frequently required in this maneuver. The procedure is also capable of recognizing when a numerical solution is only suboptimal and a powered flyby could improve the mission.

Introduction

THE optimization of interplanetary missions is a difficult problem inasmuch as the mission is partially performed inside the sphere of influence of planets and minor bodies in the solar system. To maximize payload mass, the mission characteristic velocity is reduced by increasing the number of planetary encounters where free gravity assist is received. The burn time of present technology engines is quite short, in comparison to the overall flight time, and the assumption of impulsive maneuvers is generally accepted. The matched-conic approximation[1] is widely used in preliminary analyses; numerical codes[2,3] for the optimization of this kind of problem are available that consider almost all of the possible constraints and objectives. The authors have recently proposed a simplified approach[4] that neglects the time spent by the spacecraft on the planetocentric hyperbolas; the application of the theory of optimal control to the two-body problem equations can, thus, be confined within the heliocentric reference frame. However, the technique takes into account that thrust impulses are usually given inside planetary spheres of influence; time constraints and unpowered flybys are also considered.

The technique has been applied to the preliminary analysis of a crewed round-trip Mars mission; in favorable circumstances, a free-height Venus flyby produces an important improvement of performance. The same technique is now applied to the trajectory of a spacecraft that is sent toward the outer planets by taking advantage of an Earth flyby. The spacecraft often grazes the Earth's atmosphere, and a minimum-height constraint must be enforced. This paper provides optimum conditions for minimum-height flyby, which are suitable for the simplified analysis in the heliocentric frame. The same conditions are useful to solve more complex problems[5] than the present numerical application; this illustrative example could obviously be analyzed by means of a simpler technique, for instance, a parameter optimization.

A powerful technique to increase the payload mass of a spacecraft was introduced by Hollenbeck,[6] who first proposed a delta velocity Earth-gravity-assist (ΔV-EGA) trajectory. A deep-space maneuver allows a spacecraft that has left the Earth to re-encounter it with a larger hyperbolic excess velocity (V_∞ leveraging) and to obtain a free gravity assist. Some interesting aspects of this class of trajectories have been pointed out by Sweetser,[7] who has analyzed the maneuver by means of Jacobi's integral in the restricted three-body problem. However, the term V_∞ leveraging and a systematic analysis of ΔV-EGA trajectories are due to Sims et al.[8] They assume that the Earth's orbit around the sun is circular and provide a clever means to find suboptimal trajectories by nullifying thrust-misalignment losses. In this paper, the authors first apply their optimization procedure to the same problem, as formulated by Sims et al.,[8] to quantify the improvement that is obtained by flying an optimal trajectory. The eccentricity of the Earth's orbit is then considered, to obtain more realistic knowledge of ΔV-EGA maneuvers.

Statement of the Problem

An exterior ΔV-EGA maneuver that sends the spacecraft toward the outer planets is considered. Similar trajectories inside the Earth's orbit can be used to reach the inner planets. The same analytical procedures can be used for both classes of missions. The spacecraft departs from an Earth parking orbit, which is assumed to be circular (185-km altitude) and in the ecliptic plane; the minimum allowable height during the flyby is 200 km. The aim of the maneuver is to minimize the total ΔV for an assigned aphelion radius or, in a dual form, to maximize the aphelion radius for an assigned total ΔV.

In the analysis proposed by Sims et al.[8] the patched-conic model is adopted; gravitational fields are, therefore, radial and accounted for by means of an inverse square law. The spacecraft is launched from the Earth with a hyperbolic excess velocity parallel to the velocity of the planet, which is assumed to move on a circular orbit around the sun. The simplest exterior trajectory is shown in Fig. 1; the vehicle is inserted into a heliocentric orbit with a period that is slightly greater than an integer number of years. At the aphelion, thrust is again used to provide a tangential velocity impulse Δv_1 that lowers the perihelion. The spacecraft re-encounters the Earth before or after the perihelion with a larger and nontangential hyperbolic excess velocity that is usefully rotated by gravitation. The corresponding trajectory is not optimal, but a simple iterative algorithm is sufficient to determine the Δv_1 impulse that is necessary for the intercept. Multiple revolutions of the Earth and spacecraft on their orbits are also possible: Trajectories are classified by means of the designation[8] $K{:}L(M)^{\pm}$ where

K = number of Earth orbit revolutions
L = number of spacecraft orbit revolutions
M = spacecraft orbit revolution on which Δv_1 is applied
$\pm$ = Earth encounter after/before spacecraft orbit perihelion

The patched-conic model is also used here; the flight time inside the Earth's sphere of influence is neglected and the trajectory analysis is limited to the heliocentric reference frame. The trajectory is, therefore, composed of a succession of ballistic arcs that are separated by points where the spacecraft velocity undergoes jumps. The

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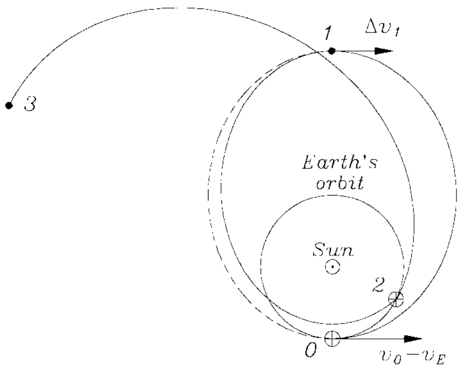

Fig. 1 Exterior ΔV-EGA trajectory.

equations of motion of a spacecraft, under only the influence of the sun's gravitational acceleration $\boldsymbol{g}(\boldsymbol{r})$, are

$$\dot{\boldsymbol{r}} = \boldsymbol{v}, \qquad \dot{\boldsymbol{v}} = \boldsymbol{g} \tag{1}$$

By applying the theory of optimal control, the Hamiltonian function is defined as

$$H = \boldsymbol{\lambda}_r^T \boldsymbol{v} + \boldsymbol{\lambda}_v^T \boldsymbol{g} \tag{2}$$

and the adjoint equations for the problem are

$$\dot{\boldsymbol{\lambda}}_r^T = -\frac{\partial H}{\partial \boldsymbol{r}} = -\boldsymbol{\lambda}_v^T G, \qquad \dot{\boldsymbol{\lambda}}_v^T = -\frac{\partial H}{\partial \boldsymbol{v}} = -\boldsymbol{\lambda}_r^T \tag{3}$$

where $G(\boldsymbol{r})$ is the gravity-gradient matrix. The procedure searches for the trajectory that achieves an assigned aphelion distance after the flyby, while minimizing the characteristic velocity

$$\Delta V = \Delta v_0 + \Delta v_1 \tag{4}$$

which is the arithmetic sum of the velocity changes that are performed by expending the propellant

$$\Delta v_1 = \left|\boldsymbol{v}_{1_+} - \boldsymbol{v}_{1_-}\right| \tag{5}$$

where $\boldsymbol{v}_{1_-}$ and $\boldsymbol{v}_{1_+}$ are the spacecraft velocity vectors just before and after the application of the deep-space impulse, and

$$\Delta v_0 = \sqrt{(\boldsymbol{v}_0 - \boldsymbol{v}_E)^2 + 2v_c^2} - v_c \tag{6}$$

where $\boldsymbol{v}_0$ is the spacecraft velocity on leaving the Earth's sphere of influence, $\boldsymbol{v}_E(t_0)$ is the planet's velocity vector, and v_c is the magnitude of the circular velocity on the Earth parking orbit. This approach allows an analysis of the ΔV-EGA maneuver by taking the eccentricity of the Earth's orbit into account and removing any simplifying assumptions concerning the position where the velocity impulse $\Delta \boldsymbol{v}_1$ is applied, its direction, and the direction of the hyperbolic excess velocity at departure $\boldsymbol{v}_0 - \boldsymbol{v}_E$.

Optimization

The differential problem is completed by boundary conditions that, in part, are provided by the theory of optimal control. The optimal conditions used to minimize ΔV are the same as in the dual problem of maximizing the aphelion radius. The missions considered in this paper are two-dimensional, but the procedure has already been applied to three-dimensional problems.[4]

At the final aphelion, where the spacecraft radial position and radial velocity component are prescribed, $\delta \boldsymbol{r}$ and $\delta \boldsymbol{v}$ are tangential vectors, and the transversality conditions[4] state that $\boldsymbol{\lambda}_r$ and $\boldsymbol{\lambda}_v$ must be radial. At the free final time $H = 0$, and one deduces from Eq. (2) that $\lambda_v = 0$.

The other necessary conditions for the optimality of a maneuver that exploits a free-height Earth flyby have been derived and described in detail in a previous paper.[4] Only the conditions concerning the velocity adjoint vector or primer vector are summarized here, due to their significance in the optimization of spacecraft maneuvers.

It is well known that the primer vector is parallel to the velocity impulse when the spacecraft uses its engine in the gravitational field of the main attractive body. The primer vector must have unit magnitude, according to prevailing literature, while its time derivative is zero, as the deep-space impulse is not time constrained.

To leave the Earth, thrust is used inside the planet's sphere of influence, and the primer vector

$$\boldsymbol{\lambda}_v = \frac{\boldsymbol{v}_0 - \boldsymbol{v}_E}{\sqrt{(\boldsymbol{v}_0 - \boldsymbol{v}_E)^2 + 2v_c^2}} \tag{7}$$

is parallel to the hyperbolic excess velocity $\boldsymbol{v}_0 - \boldsymbol{v}_E$. Equation (7) also prescribes the primer magnitude that is lower than unity and depends only on the hyperbolic excess velocity and radius of the parking orbit. The departure date is unconstrained and the primer magnitude is stationary (that is, its time derivative is zero).

The primer vector and the hyperbolic excess velocity must be parallel soon before and after the free-height flyby that, according to the assumed model, instantaneously rotates the hyperbolic excess velocity. The primer magnitude and its time derivative must also be continuous. In some problems, the optimal solution could imply an approach that is too close to the planet; a further constraint concerning the minimum admissible radial distance from the Earth's surface must be added, and different optimum conditions are necessary.

Minimum-Height Flyby

The hyperbola perigee is, in this case, fixed at the minimum allowable value $r_{\min}$; the corresponding circular velocity $v_p = \sqrt{(\mu / r_{\min})}$ is, therefore, an assigned constant. The hyperbolic excess velocity $\boldsymbol{v}_\infty = \boldsymbol{v}_2 - \boldsymbol{v}_E$ is introduced for the sake of conciseness; subscript 2 denotes the re-encounter with the Earth; further subscripts $-$ and $+$ are used to distinguish between values just before and after the flyby. The half-angle of the hyperbola ϕ appears only as a function of v_{∞_-}; one easily obtains

$$\cos\phi = \frac{v_p^2}{\boldsymbol{v}_{\infty_-}^2 + v_p^2}, \qquad \frac{\mathrm{d}\phi}{\mathrm{d}\boldsymbol{v}_{\infty_-}} = \frac{2}{\tan\phi}\frac{\boldsymbol{v}_{\infty_-}}{v_{\infty_-}^2 + v_p^2} \tag{8}$$

and

$$A = \boldsymbol{v}_{\infty_-}\frac{\mathrm{d}\phi}{\mathrm{d}\boldsymbol{v}_{\infty_-}} = \frac{2}{\tan\phi}\frac{v_{\infty_-}^2}{v_{\infty_-}^2 + v_p^2}, \qquad B = \cos 2\phi - A \sin 2\phi \tag{9}$$

are conveniently defined.

The position constraints $\boldsymbol{r}_{2_+} = \boldsymbol{r}_{2_-} = \boldsymbol{r}_E(t_2)$ are the same as in a free-height flyby, and $\boldsymbol{\lambda}_{r_+} - \boldsymbol{\lambda}_{r_-} = \boldsymbol{\mu}_2$ is again found (the position adjoint vector is discontinuous). Two scalar constraints

$$\boldsymbol{v}_{\infty_+}^2 = \boldsymbol{v}_{\infty_-}^2, \qquad \boldsymbol{v}_{\infty_+} \cdot \boldsymbol{v}_{\infty_-} = -\cos 2\phi \boldsymbol{v}_{\infty_-}^2 \tag{10}$$

concern the spacecraft velocity, and the scalar multipliers μ_3 and μ_4 are associated with these. The application of the transversality conditions[4] provides

$$\boldsymbol{\lambda}_{v_-} - 2\mu_3 \boldsymbol{v}_{\infty_-} + \mu_4 \boldsymbol{v}_{\infty_+} + 2\mu_4 B \boldsymbol{v}_{\infty_-} = 0 \tag{11}$$

$$-\boldsymbol{\lambda}_{v_+} + 2\mu_3 \boldsymbol{v}_{\infty_+} + \mu_4 \boldsymbol{v}_{\infty_-} = 0 \tag{12}$$

By carrying out the cross products between Eqs. (11) and (12) and vectors $\boldsymbol{v}_{\infty_-}$ and $\boldsymbol{v}_{\infty_+}$, respectively, one deduces

$$\boldsymbol{\lambda}_{v_-} \times \boldsymbol{v}_{\infty_-} = -\mu_4 \boldsymbol{v}_{\infty_+} \times \boldsymbol{v}_{\infty_-} \tag{13}$$

$$\boldsymbol{\lambda}_{v_+} \times \boldsymbol{v}_{\infty_+} = \boldsymbol{\lambda}_{v_-} \times \boldsymbol{v}_{\infty_-} \tag{14}$$

Two important conclusions arise: The primer vector must be in the plane of the flyby just before and after it; this is shown by the dot product between Eq. (14) and either vector $\boldsymbol{v}_{\infty_-}$ or $\boldsymbol{v}_{\infty_+}$. Moreover, if $\lambda_v^{\parallel}$ and $\lambda_v^{\perp}$ denote the primer components that are respectively parallel and orthogonal to $\boldsymbol{v}_\infty$, Eq. (14) also implies $\lambda_{v_+}^{\perp} = \lambda_{v_-}^{\perp}$.

One should note that the vectorial Eq. (14) also prescribes the direction of the components $\lambda_v^{\perp}$: Both components are correctly directed toward the Earth in Fig. 2. The primer is suggesting that the trajectory would be improved with a different direction of the arrival hyperbolic excess velocity or with a larger turn angle produced by the flyby, that is, for instance, with a lower perigee altitude. The minimum-height constraint must be removed when both the components $\lambda_v^{\perp}$ have the reverse directions; $\boldsymbol{\lambda}_v$ becomes parallel to $\boldsymbol{v}_\infty$, i.e., $\lambda_v^{\perp} = 0$, in the optimal solution with a free-height flyby.

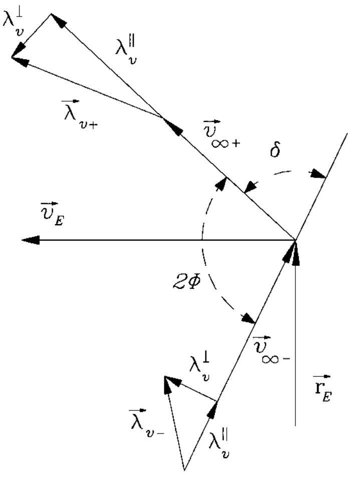

Fig. 2 Primer vector for optimal minimum-height flyby.

Before deriving the last condition, Eq. (13) is written in the form

$$\lambda_{v_-}^\perp = -\mu_4 v_{\infty-} \sin 2\phi \tag{15}$$

If the dot products between Eqs. (11) and (12) and, respectively, vectors $\boldsymbol{v}_{\infty-}$ and $\boldsymbol{v}_{\infty+}$ are carried out, one obtains

$$\lambda_{v_-}^\parallel v_{\infty-} - 2\mu_3 v_{\infty-}^2 + \mu_4 \boldsymbol{v}_{\infty+}\cdot\boldsymbol{v}_{\infty-} + 2\mu_4 B v_{\infty-}^2 = 0 \tag{16}$$

$$-\lambda_{v_+}^\parallel v_{\infty+} + 2\mu_3 v_{\infty+}^2 + \mu_4 \boldsymbol{v}_{\infty-}\cdot\boldsymbol{v}_{\infty+} = 0 \tag{17}$$

which are added to provide, using Eqs. (10) and (15),

$$\lambda_{v_+}^\parallel = \lambda_{v_-}^\parallel + 2A\lambda_{v_-}^\perp \tag{18}$$

Equations (10) and (18), together with the continuity of $\lambda_v^\perp$, constitute four conditions that allow one to solve the discontinuities of $\boldsymbol{v}_2$ and $\boldsymbol{\lambda}_v$ that arise from the two-dimensional flyby. In three-dimensional problems, two further conditions prescribe the nullity of the $\boldsymbol{\lambda}_{v_-}$ and $\boldsymbol{\lambda}_{v_+}$ components that are outside the flyby plane.

The necessary transversality conditions that concern the Hamiltonian provide

$$H_{2_+} - H_{2_-} = \boldsymbol{\mu}_2^T \boldsymbol{v}_E + \Big[2\mu_3\big(\boldsymbol{v}_{\infty+} - \boldsymbol{v}_{\infty-}\big) + \mu_4\big(\boldsymbol{v}_{\infty+} + \boldsymbol{v}_{\infty-}\big) + 2\mu_4 B \boldsymbol{v}_{\infty-}\Big]^T \dot{\boldsymbol{v}}_E \tag{19}$$

which, by means of Eqs. (11) and (12), is easily reduced to the form

$$H_{2_+} - H_{2_-} = \big(\boldsymbol{\lambda}_{r_+} - \boldsymbol{\lambda}_{r_-}\big)^T \boldsymbol{v}_E + \big(\boldsymbol{\lambda}_{v_+} - \boldsymbol{\lambda}_{v_-}\big)^T \dot{\boldsymbol{v}}_E \tag{20}$$

By using Eq. (2) and considering that the Earth also obeys Eqs. (1), one obtains

$$\boldsymbol{\lambda}_{r_+}^T \boldsymbol{v}_{\infty+} - \boldsymbol{\lambda}_{r_-}^T \boldsymbol{v}_{\infty-} = 0 \tag{21}$$

which is formally the same as in a free-height flyby. The time derivative of the primer magnitude is now discontinuous inasmuch as the primer vector and hyperbolic excess velocity are no longer parallel.

Numerical Results

The state and adjoint differential equations, together with the relevant boundary conditions, constitute a boundary value problem, which is solved by means of a shooting procedure[9] that exploits the numerical integration of the sensitivity equations. The numerical efficiency is improved by a particular treatment of the interior points, which allows for the comprehensive optimization of the whole trajectory, instead of matching arcs that are separately optimized.

The problem is first solved by considering a free-height Earth flyby. The solution is then inspected: If the minimum-altitude constraint is violated, a new solution is sought by imposing the optimum necessary conditions for the minimum-height flyby. Finally, Pontryagin's maximum principle is used to check whether the solution is optimal; in some cases, this further inspection indicates that the maneuver could be improved by a powered flyby. These cases cannot, at the moment, be solved using the optimization procedure.

It is necessary to note that the theory of optimal control provides necessary conditions for a local optimum. Interplanetary missions are very complex, mainly because of the possibility of exploiting flybys with the minor bodies in the solar system. This paper is aimed at looking for local optima; for instance, a 2:1(1)$^-$ ΔV-EGA mission with a single unpowered flyby is considered and optimized in the following. The authors are aware that in many cases a different concept of mission can provide a better performance; therefore, they have explicitly declared the terms of the problem they solve.

Circular Earth orbit

The procedure is first applied by assuming a circular motion of the Earth around the sun, to quantify the benefit of an optimal trajectory with respect to the suboptimal solution that Sims et al.[8] suggested. Figure 3 shows the true anomalies ν of the spacecraft at launch and at the deep-space impulse on the elliptic orbit joining these points; the thrust misalignment α, i.e., the angle between $\boldsymbol{v}_{1_-}$ and $\Delta\boldsymbol{v}_1$, is also shown ($\alpha > \pi$ means that the radial thrust component points to the sun); the suboptimal solution is obtained by assuming $\nu_0 = 0$ and $\nu_{1_-} = \alpha = \pi$. A value of the characteristic velocity ΔV is found, where the optimal solution does not differ from the suboptimal strategy. This is more evident in Fig. 4, which presents the increment in the final aphelion radius that is provided by optimization. Three different regions are encountered by increasing ΔV: A free-height flyby (FHFB) is exploited in the first region, but the perigee radius is progressively reduced until the minimum-height constraint must be enforced; in the minimum-height flyby (MHFB) region the maneuver used by Sims et al.[8] is, just in one case, optimal. For high characteristic velocities, the primer magnitude becomes larger than

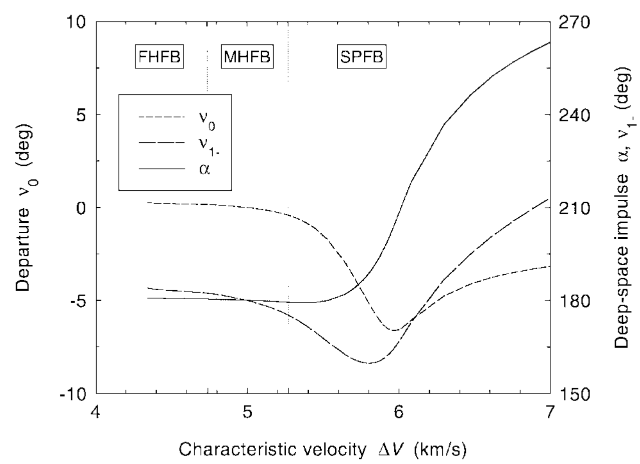

Fig. 3 Features of optimal 2:1(1)$^-$ trajectories.

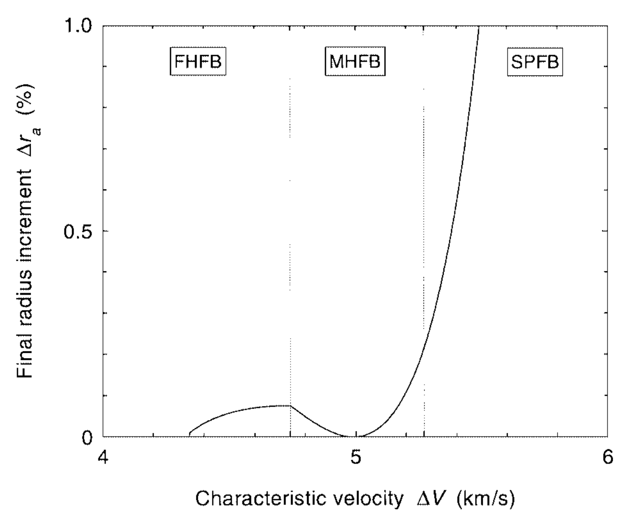

Fig. 4 Increment of the aphelion radius provided by optimization.

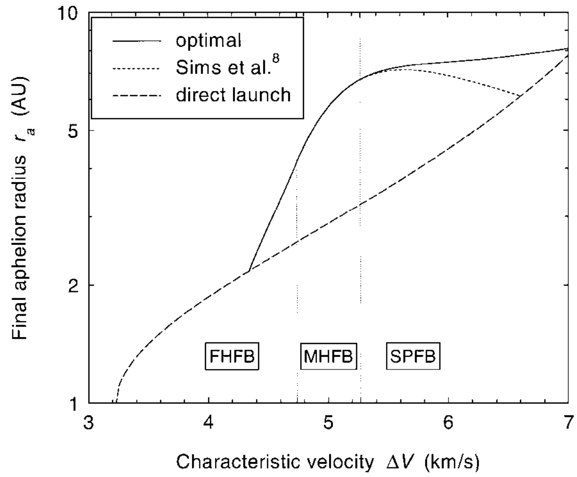

Fig. 5 Comparison of 2:1(1)⁻ maneuvers (circular Earth orbit).

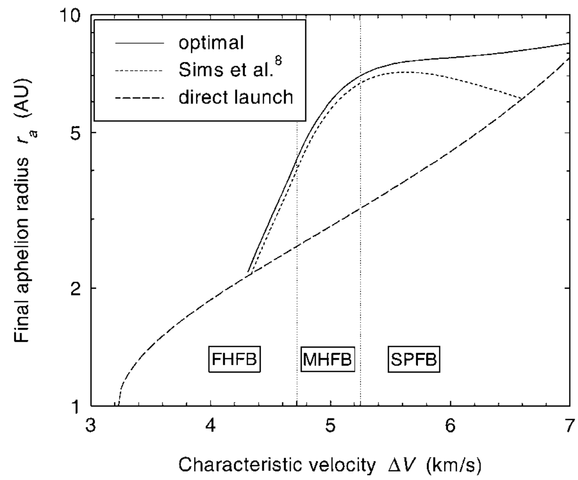

Fig. 7 Optimal 2:1(1)⁻ trajectories (elliptic Earth orbit).

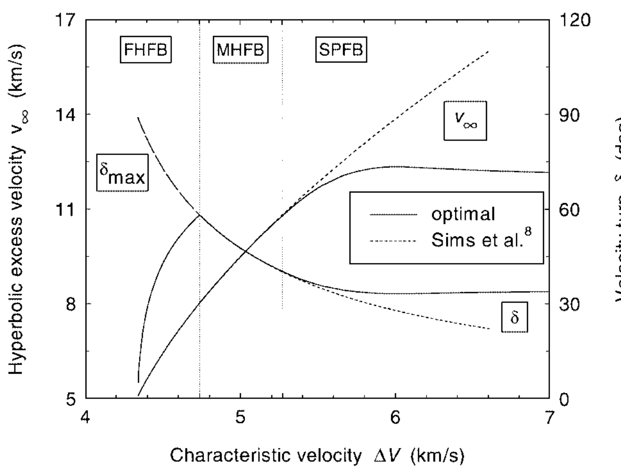

Fig. 6 Comparison of 2:1(1)⁻ maneuvers (circular Earth orbit).

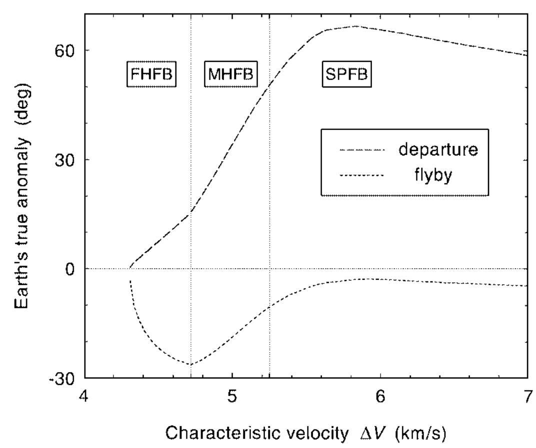

Fig. 8 Earth's position for optimal departure and flyby.

the limit value for optimality, when the spacecraft is leaving the Earth after the flyby. The suggested powered-flyby (SPFB) region comprises these cases, which would require the combined use of propulsion and gravity assist; thrust should be applied on the arrival hyperbola after the passage by the perigee; a small misalignment, with respect to the spacecraft velocity, increases the turn angle and seems to be advisable.[10]

This optimization procedure is not able to deal with powered flybys, and the achieved solutions can be considered to be optimal just inside the limits of the assumed switching structure, i.e., the order and type of the discontinuities in the spacecraft heliocentric trajectory. The largest improvements are, however, obtained in these cases (Fig. 5), but the outcome is not surprising, as the final aphelion distance cannot be a decreasing function of ΔV. The optimal strategy for high-ΔV missions is shown in Fig. 6. The use of the minimum-height flyby begins when the hyperbola turn angle δ attains the maximum admissible value $\delta_{\max} = \pi - 2\phi$, which depends only on v_∞, according to Eq. (8). A further increment of the characteristic velocity causes a larger v_∞ but a smaller δ. Whereas v_∞ is an increasing function of ΔV in the suboptimal solution, the optimal strategy eventually maintains v_∞ almost constant to avoid an excessive reduction of δ.

Elliptic Earth Orbit

The procedure is then applied to a more realistic model that takes the eccentricity of the Earth's orbit into account. In particular, the varying velocity of the Earth on its orbit can be profitably used, and the procedure suggests the optimal strategy. Figure 7 presents the corresponding results; the benefit is evident and is mainly (almost 99%) due to the eccentricity of the Earth's orbit.

The true anomalies of the Earth at departure and flyby are presented in Fig. 8. Performance is improved if the spacecraft leaves and re-encounters the Earth in the proximity of the perihelion to take the greatest advantage from the planet's velocity. An increasing phase angle between departure and gravity assist is required to reinforce the V_∞ leveraging and achieve larger values of the final radius. Initially, in the FHFB region, the Earth positions at launch and flyby are equally important; the maximum benefit is obtained if these positions are almost at the same distance from the Earth's orbit perihelion. When the maneuver requires an MHFB (MHFB and SPFB regions), this event becomes more critical than departure and moves closer to the perihelion, while the needed phase-angle is provided by a larger true anomaly of the Earth at launch.

Conclusions

A simple technique for the preliminary analysis of interplanetary missions has been used to study ΔV-EGA trajectories. The technique applies the theory of optimal control to two-body problem equations in the heliocentric reference frame. A minimum-height constraint is often required in these missions; the necessary optimum conditions for the constrained flyby have been derived and successfully tested. The assumption of a circular Earth orbit has permitted a comparison with suboptimal results provided by an efficient procedure that has recently been proposed. For low characteristic velocities, the suboptimal strategy is barely improved by optimization. A great benefit is instead obtained for high-ΔV missions, but the new trajectories can be considered optimal only if a powered flyby is excluded a priori. Moreover, a better performance (with a longer time length) could be achieved by other classes of ΔV-EGA maneuvers or multiple flybys. This optimization procedure instead allows preliminary analyses that take the eccentricity of the Earth's orbit into account. More realistic estimates, with indications concerning the optimum departure dates, are provided by the proposed procedure, which is much handier than other sophisticated codes based on the theory of optimal control; in particular, a wide analysis of the problem can be carried out in a brief time by using a low-performance workstation.

References

[1]Glandorf, D. R., "Primer Vector Theory for Matched-Conic Trajectories," *AIAA Journal*, Vol. 8, No. 1, 1970, pp. 155, 156.

[2]Friedlander, A., "(MULIMP) Multi-Impulse Trajectory and Mass Optimization Program," Science Application International Corp., Report No. SAI-120-383-14, San Diego, CA, April 1974.

[3]Sauer, C. G., "MIDAS—Mission Design and Analysis Software for the Optimization of Ballistic Interplanetary Trajectories," *Journal of the Astronautical Sciences*, Vol. 37, No. 3, 1989, pp. 251–259.

[4]Casalino, L., Colasurdo, G., and Pastrone, D., "Optimization Procedure for Preliminary Design of Opposition-Class Mars Missions," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, 1998, pp. 134–140.

[5]Casalino, L., Colasurdo, G., and Pastrone, D., "Optimal Low-Thrust Trajectories Using Flyby," 8th AAS/AIAA Space Flight Mechanics Meeting, AAS Paper 98-200, Monterey, CA, Feb. 1998.

[6]Hollenbeck, G. R., "New Flight Techniques for Outer Planet Missions," AAS/AIAA Astrodynamics Specialist Conf., American Astronautical Society, AAS Paper 75-087, Nassau, Bahamas, July 1975.

[7]Sweetser, T. H., "Jacobi's Integral and ΔV-Earth-Gravity-Assist (ΔV-EGA) Trajectories," AAS/AIAA Astrodynamics Specialist Conf., American Astronautical Society, AAS Paper 93-635, Victoria, BC, Canada, Aug. 1995.

[8]Sims, J. A., Longuski, J. M., and Staugler, A. J., "V_∞ Leveraging for Interplanetary Missions: Multiple-Revolution Orbit Techniques," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 3, 1997, pp. 409–415.

[9]Colasurdo, G., and Pastrone, D., "Indirect Optimization Method for Impulsive Transfer," *Proceedings of the AIAA/AAS Astrodynamics Conference*, AIAA, Washington, DC, 1994, pp. 441–448 (AIAA Paper 94-3762).

[10]Colasurdo, G., Casalino, L., and Pastrone, D., "Simple Strategy for Powered Flyby," 48th International Astronautical Congress, International Astronautical Federation, Paper IAF-97-A.6.03, Torino, Italy, Oct. 1997.